Physics C161 Final Project

Claire Chen

# Designing a Receiver for the Cosmic Microwave Background

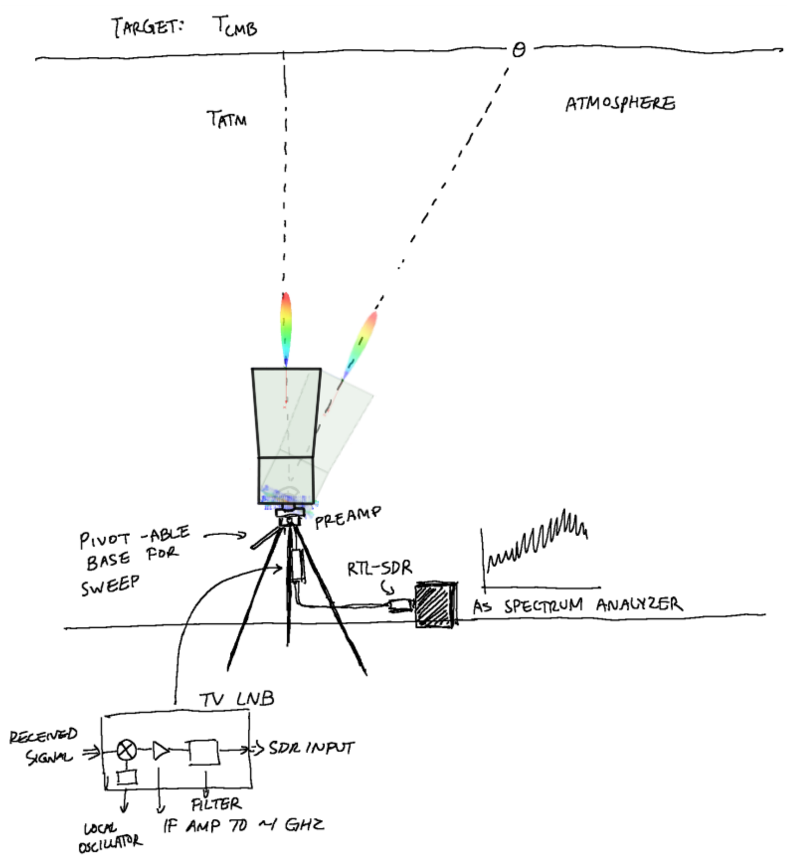


Figure 1: The end result: my design for an instrument for measuring the temperature of the CMB at 19.1 GHz, that can be constructed using amateur radio hardware and a sheet metal horn antenna.

## Abstract

We've extensively discussed the cosmic microwave background (CMB) in this class and how powerful it is as a tool in understanding the development of the universe. Being able to detect, measure, and map the CMB is thus very important, and understanding how we do this is what I am interested in for this project. In this project, I aim to gain a better understanding

of hardware design for cosmology, by designing an instrument for measuring the temperature of the CMB that could be constructed as simply and cheaply as possible, from the ground up. The final design I have is based around the RTL-SDR, an inexpensive ($30) software-defined radio, and a horn antenna designed for 19-21 GHz. I designed an antenna for this application and simulated its radiation patterns, and outlined a process for how to measure the CMB temperature with this instrument, including how I would account for atmospheric absorption and emission and noise. I was not able to actually construct this setup and carry out this process, but the design is something that I could realistically build with tools and components that I can access at Berkeley. With this starting point, I compared my design and process to what Penzias and Wilson actually used for the first detection of the CMB.

# 1 Theory and Design

Our goal is to design an instrument that can detect the CMB and measure its temperature, that will be as simple as possible to construct. From our knowledge of thermodynamics, if we can measure the power of the radiation emitted by a blackbody, and we know the temperature of an antenna that is receiving this radiation, we should be able to calculate the temperature of the blackbody. Since the CMB is blackbody radiation, we can design an instrument that uses this principle to measure the temperature of the CMB. Here is what we'll need in order to determine the temperature of the CMB:

1. We need to design an antenna that can receive the CMB, taking into account that we will have to deal with emissions from other microwave sources. To design the antenna, we'll need to choose a frequency to observe.
2. If we know the power received, we can determine the temperature of the blackbody source from Planck's law.
3. We need to measure the power of the signal received by the antenna. This could be done with a cheap software-defined radio (SDR) acting as a spectrum analyzer.
4. We need to know the contribution to the power that would be from sources other than the CMB. For example, as radiation travels through the atmosphere, we'll need to take into account atmospheric absorption and emissions. Another source is noise from the receiver's electronics.
5. We can account for the atmosphere by sweeping the antenna over a range of angles and seeing how the power measured changes. This will let us calculate the temperature contribution from the atmosphere when the antenna is in position to observe, and we can then subtract this out from our measured overall power.
6. We can account for the effect of the noise from electronics by calibrating the receiver electronics at different temperatures.
7. We can then determine the temperature of the CMB, given the total power we receive, the temperature contribution from the atmosphere, the temperature contribution from noise, and the gain of the antenna.

Let's get into the specifics of how we'll do each of these steps, starting with determining the frequency.

## 1.1 Choosing a Frequency

Our starting point is Planck's law for blackbody radiation, which gives us the spectral energy density at some frequency. In other words, this will tell us how much electromagnetic energy we have in a given range of frequencies, in a given volume.

For photons to be in thermal equilibrium at temperature $T$, the spectral energy density at frequency $\nu$ is given by:

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \propto \frac{\nu^3}{e^{\beta h\nu} - 1} \tag{1}$$

From lecture, we have $T_{CMB} = 2.725K$. Using Planck's law we find that the range of frequencies for the CMB is quite large, from around 10 GHz to 400 GHz. The peak is at 160.2 GHz, so one might think that we should design our instrument for receiving this frequency. However, it is not so simple: we are observing within the atmosphere, and the atmosphere is only semi-transparent to microwaves. Oxygen, water vapor, nitrogen, and other gases in the atmosphere all attenuate (scatter and absorb) microwave radiation between around 30 GHz to 210 GHz,[1] which will make it much more difficult to receive and identify the CMB in this range. So, looking at the peak is not an option; we'll have to try and observe at the lower end of our frequency range. In that case, we will also need to consider potential interference from other sources: for example, cell phones and GPS operate in the L band below 2 GHz, and most satellites operate in the C, X, and Ku bands, so between 2 and 18 GHz. Since water vapor has a significant absorption line at 22 GHz,[2] I chose to design for 20 GHz.

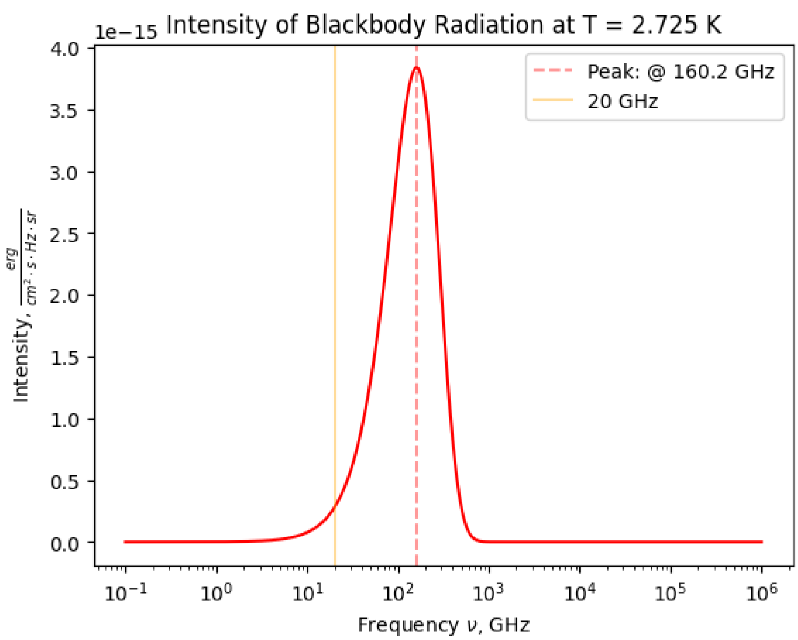


Figure 2: Modeling CMB as blackbody radiation, with temperature of 2.725 K

## 1.2 Power and Temperature

We can use the Rayleigh-Jean approximation, as the CMB should be low enough frequency for this to be consistent with Planck's law. Basically, if we Taylor expand $e^{\frac{h\nu}{k_B T}} \approx 1 + \frac{h\nu}{k_B T}$, Planck's law becomes this:

[1]P. Langley. "Absorption of Microwaves by Atmospheric Gases". In: *Atmospheric Remote Sensing by Microwave Radiometry.* 1993.

[2]Ibid.

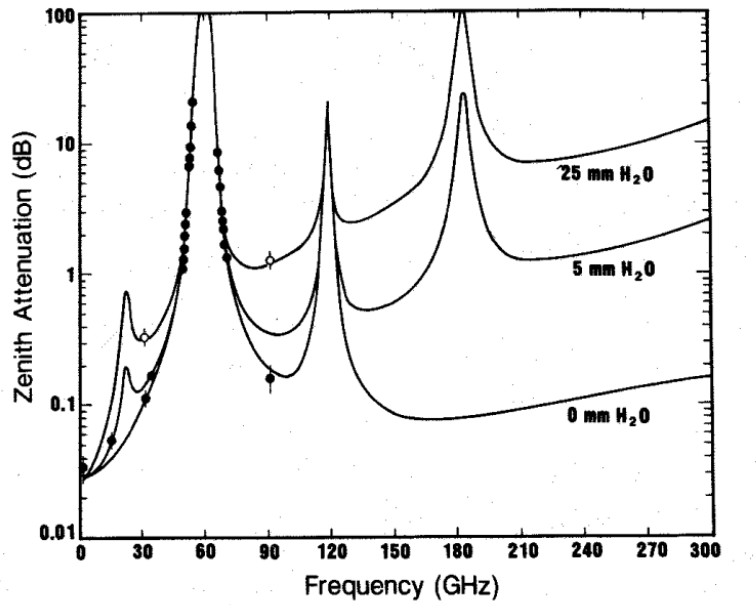


Figure 3: From Rosenkrantz, 1993, showing how various atmospheric gases attenuate in the frequency range we're interested in.

$$B(\nu, T) = \frac{2\pi\nu^2 k_B T}{c^2} \propto T \tag{2}$$

So, if we fix our frequency (20 GHz), we have a nice linear relationship between the temperature of the blackbody and the intensity of the radiation, which means that we can also relate the blackbody temperature $T_{BB}$ to the power our antenna receives, $P_{REC}$. Taking into account that the antenna will have a gain of $G$:

$$P_{REC} = GT_{BB} \propto T_{BB} \tag{3}$$

This relation seems simple enough, but our setup won't only be picking up the CMB. The power we measure will also include contributions from the atmosphere absorbing and emitting radiation, and from thermal noise from the electronics:

$$P_{REC} = (P_{CMB} + P_{ATM}) + P_{NOISE} \tag{4}$$

$$\frac{P_{REC}}{G} - T_{NOISE} = T_{CMB} + T_{ATM} \tag{5}$$

So, this is how we will calculate the temperature of the CMB with this instrument:

$$T_{CMB} = \frac{P_{REC}}{G} - T_{ATM} - T_{NOISE} \tag{6}$$

### 1.3 Determining $T_{NOISE}$ and $T_{ATM}$

To determine $T_{NOISE}$, we know that it comes from the fact that electrons moving in a conductor have an associated thermal energy. From Planck's law, this means there will

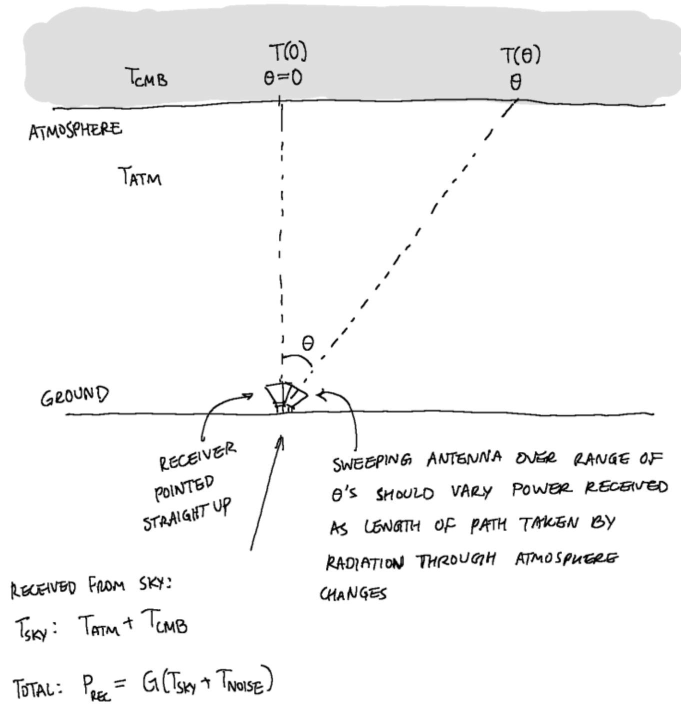


Figure 4: Accounting for the atmosphere. We want to find the temperature contribution of the atmosphere when the antenna is pointed straight up. Then, we can subtract this from the power we measure in this orientation.

be some power measured that comes from this, that is directly dependent on temperature. This is called Johnson-Nyquist noise and is impossible to fully avoid, as it follows from the laws of thermodynamics. However, it can be accounted for: since the power contributed is temperature dependent, if we have the instrument pointed directly at sources of different temperatures, the total power received will be from just Johnson-Nyquist noise and whatever the constant-temperature source is radiating.

$$P_{REC} = GT_{SOURCE} + GT_{NOISE} \tag{7}$$

$$T_{NOISE} = \frac{P_{REC}}{G} - T_{SOURCE} \tag{8}$$

So if we have several values of $T_{SOURCE}$, we could extrapolate to see what $T_{NOISE}$ will be for different observed powers, and thus find $T_{NOISE}$ experimentally.

Ultimately, we will be collecting data with the antenna pointed straight up, but we cannot determine $T_{ATM}$ from having the antenna vertical. To find $T_{ATM}$ and thus account for contributions from the atmosphere, the key fact is that the length of the path taken by radiation passing through the atmosphere to the antenna will vary depending on the angle the antenna is pointing at, and thus the power measured will vary. To do this characterization, we can sweep the antenna over a range of angles from the vertical, measure the power, and calculate a temperature. We can find $T_{ATM}(0)$ from this data and that will be the temperature contribution from the atmosphere when the antenna is pointed straight up and collecting the data we want.

$$T_{ATM}(\theta) = T_{ATM}(0)sec(\theta) \tag{9}$$

We can't measure $T_{ATM}$ in isolation, but we do have $T_{REC}$ and $T_{NOISE}$, with $T_{REC}$ including radiation from the atmosphere and CMB. Thus, the contribution from the atmosphere, which we can then subtract out, is:

$$T_{ATM}(0) = [(T_{REC}(\theta) - T_{NOISE}) - (T_{REC}(0) + T_{NOISE})]cos(\theta) \tag{10}$$

$$T_{ATM}(0) = [T_{REC}(\theta) - T_{REC}(0)]cos(\theta) \tag{11}$$

If we follow this procedure, we can then substitute into Equation (6) and find $T_{CMB}$.

## 1.4 Antenna Choice

The signal we want to detect is fairly weak, over a broad band, and originating from over the whole sky. Thus, for our receiver, we will need an antenna that is very sensitive to microwave frequencies, that ideally can operate over a range of frequencies. In addition, we would want to avoid losses and noise from scattering, and minimize the reception of environmental noise. In more specific terms, we want good directivity, meaning the power we receive from a specific direction is high compared to how we receive radiation coming from all directions. Taking all of this into account, the best antenna for our setup would be a horn antenna. How to design the antenna will be discussed in detail below, but in brief, after running some simulations, I found that a pyramidal horn antenna specifically would be a good choice.

## 1.5 Receiver Electronics

In order to process the received signal so that the power can be determined, we will need more than just the antenna. First, we will need to amplify the signal significantly while simultaneously keeping noise to a minimum, as the signal will be weak. I can then use the RTL-SDR, an extremely cheap software defined radio module, as a super cheap spectrum analyzer, to determine the received power. (There are many open-source tools already made for using the RTL-SDR as a spectrum analyzer; QSpectrumAnalyzer is a good example.)

The problem is that the RTL-SDR can receive at most 1.75 GHz,[3] depending on the model, so we will have to convert our signal to be below 1.75 GHz. We can do this with a combination of a mixer, local oscillator, amplifier, and intermediate-frequency amplifier. Alternatively, if we can find a low-noise block downconverter, or LNB, that can convert from the Ka band to L band, that could also work. These are often found in satellite TV receivers and basically include that set of components in a single package. In either case, the electronics for this instrument will look like this:

[3] *"About RTL-SDR"*. 2022. URL: https://www.rtl-sdr.com/about-rtl-sdr/.

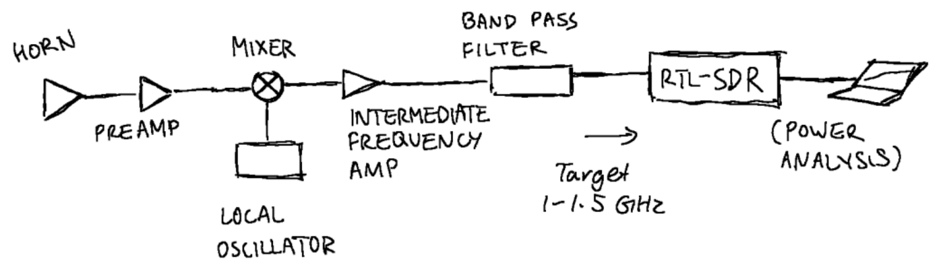


Figure 5: Block diagram.

# 2 Designing the Antenna

If we want to make a horn antenna, there are a few ways we can do it. Horn antennas come in different shapes: we could make a conical antenna, an exponentially tapered antenna, or a pyramidal antenna. Exponential horns are great for performance, but designing for this shape is more difficult, and they are much harder to construct well. Thus, we'll focus on conical and pyramidal antennas for this project.

A horn antenna has two parts: the horn itself and a waveguide. The dimensions of the horn will affect how much gain and internal reflection our antenna has at our target frequency. The dimensions of the waveguide will determine the cutoff frequency, $f_{min}$, of our antenna, or the minimum frequency that can propagate through it. In designing our antenna, we want to optimize it such that we have as much gain as possible while minimizing the amount of internal reflection. There are equations for determining the dimensions for an optimal antenna of each type for some wavelength and antenna length, and for the size of the waveguide, and we will use those below.

I then used Ansys HFSS, a finite element solver for electromagnetic fields, to simulate how each kind of antenna might perform. What I specifically wanted to learn was if the antenna will work as a receiver for a distant source like the CMB, so my model uses a far-field source approximated as an infinitely-large sphere around the antenna. I then plot the electric and magnetic field strengths as well as radiation patterns in the antenna as the receiver is excited. We can then see if the antenna has good gain and good directionality: we do not want it to be picking up signals from the environment coming from all directions instead of the CMB. The simulation will also be able to determine the efficiency of the antenna, as gain is normalized and thus it could be possible that the antenna has decent gain but low efficiency. As I was running the simulations, however, I encountered a lot of problems with mesh resolution (the student version heavily limits what you can do) which also limited the size of the antennas I could model, but the results should still be fine for a comparison.

## 2.1 Conical Horn Antenna

For an optimal conical antenna as defined above, if the slant length of the cone is $L$, then the diameter of the aperture of the cone, $d$, will be:

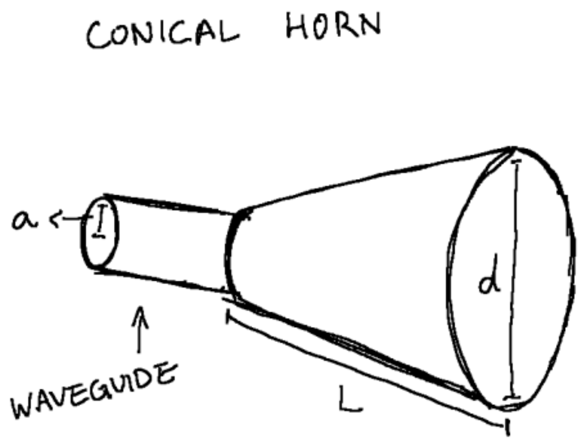


Figure 6: Conical horn antenna with key dimensions.

$$d = \sqrt{3\lambda L}\text{[4]} \tag{12}$$

The cylindrical waveguide for this antenna will have a cutoff frequency of $f_{min} = \frac{1.8412c}{2\pi a}$, where $a$ is the radius of the waveguide section.[5]

Choosing a cutoff frequency of 10 GHz and receiving frequency of 20 GHz, and using these equations as a guideline for dimensions over several iterations, I made a model of this antenna.

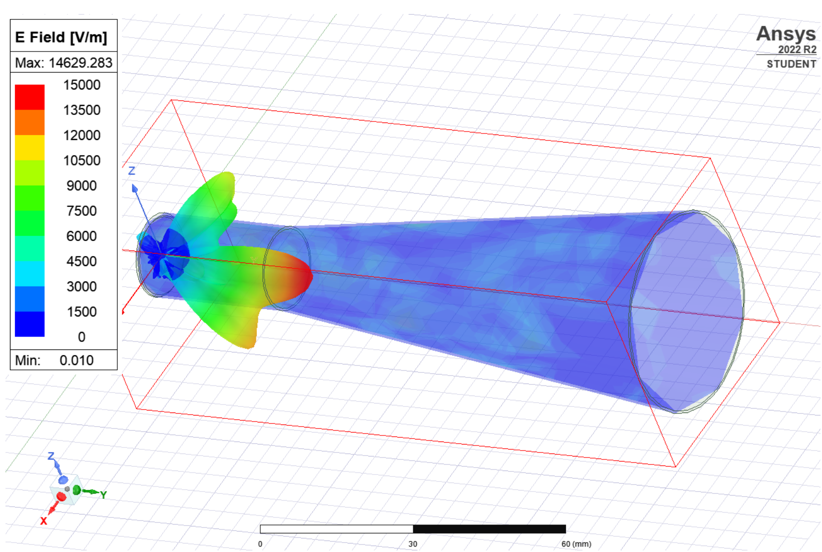


Figure 7: Model of the conical horn antenna, showing the excitation on the waveport from a far-field signal, and the magnitude of the E field over the antenna. This has the gain as an overlay, and it is clear that it is not as directional as we would want it to be.

[4] S. Silver. *Microwave Antenna Theory and Design*. Massachusetts Institute of Technology. Radiation Laboratory Series. no. 12. McGraw-Hill Book Company, 1949.

[5] Ibid.

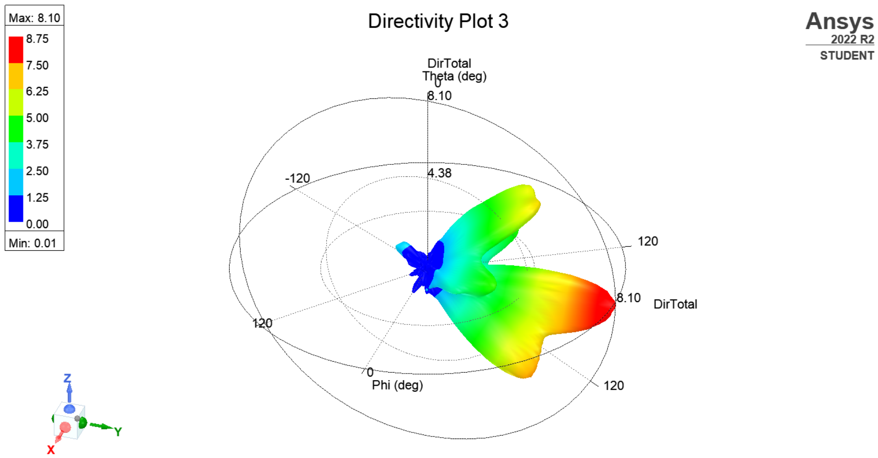


Figure 8: Directivity for the cone antenna.

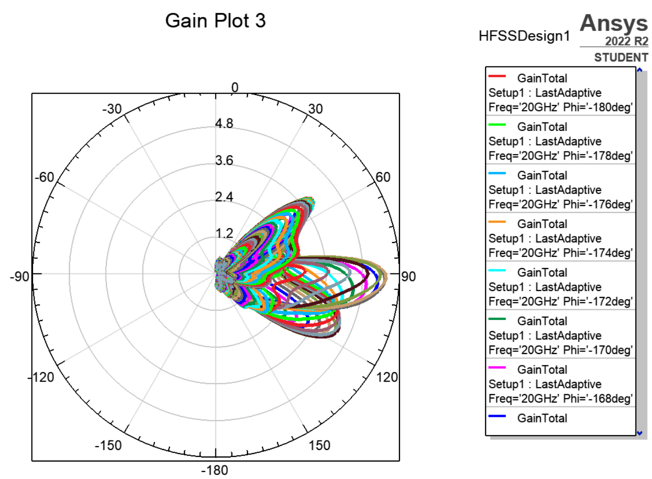


Figure 9: Radiation pattern for the cone antenna.

From the gain and directivity plots, this cone antenna would probably not be ideal for measuring the CMB. It is not as directional as we would like, with multiple lobes instead of one clear and prominent lobe, and the side lobes are concerning. The highest power density may still be through the y-axis, ie. the direction the antenna would be pointing in, but there could be significant contributions to the power from other directions that could make it much harder for the signal from the CMB to be received and identified.

One significant limitation I ran into while running the simulation was my student version of Ansys HFSS is limited in the number of elements I can use to represent the antenna. Since it is a finite element solver, the poor resolution could mean that with further adjustment, the radiation pattern and directivity of the cone antenna may not actually be this bad. Since I was stuck with this restriction though, I decided to move on to modeling a pyramidal horn antenna and see if that would have better results.

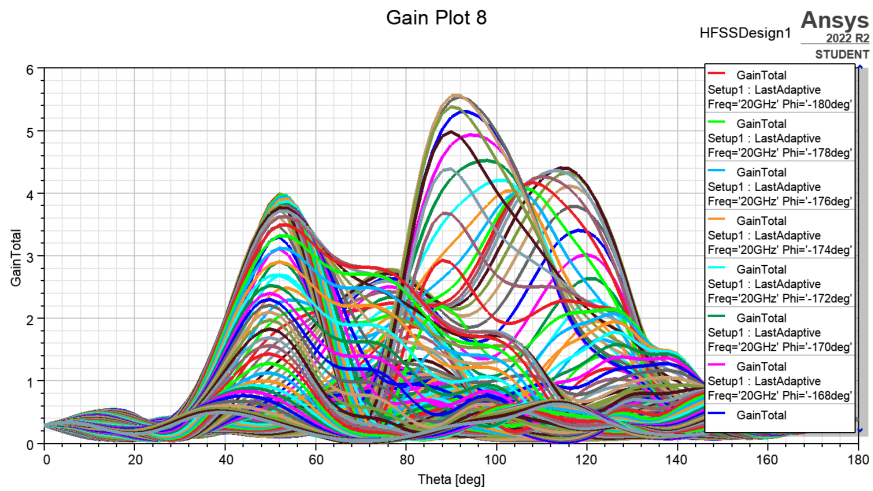


Figure 10: Plot of the gain for the cone antenna showing multiple peaks. Ideally, we should see one prominent peak in the direction the antenna is pointed in, but instead here we see several peaks spread out over a range of directions, which could mean the antenna might be more susceptible to other signals from the environment.

## 2.2 Pyramidal Horn Antenna

The corresponding design equations for the optimal pyramidal horn are similar, but have to consider the lengths of the aperture in the directions of both the electric and magnetic fields, $aE$ and $a_B$ respectively:[6]

$$a_E = \sqrt{2\lambda L_E} \tag{13}$$

$$a_B = \sqrt{3\lambda L_B} \tag{14}$$

$L_E$ and $L_B$ are the lengths of the edges in the E- and B-field directions. Similarly, for the dimensions of the waveguide section, we can find that using the cutoff frequency equation, $f_{min} = \frac{c}{2a}$, where $a$ is the length of the waveguide "box" in the E-field direction.[7]

After several iterations, this is the final design for the pyramidal horn antenna.

[6] Silver, *Microwave Antenna Theory and Design*.
[7] Ibid.

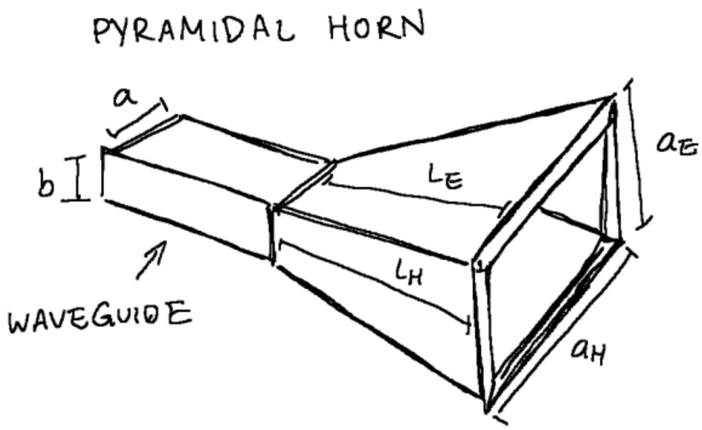


Figure 11: Pyramidal horn antenna with key dimensions.

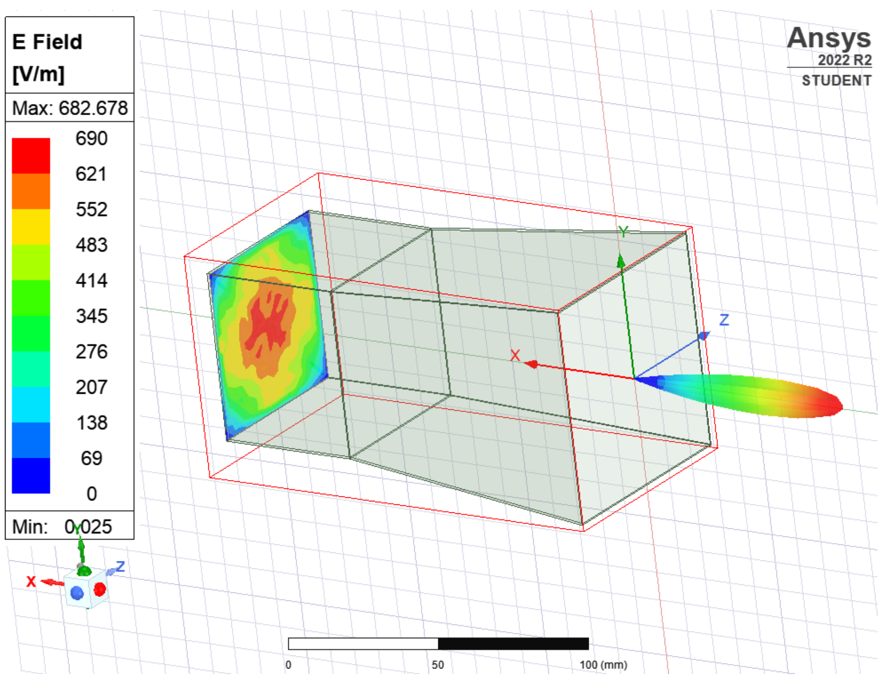


Figure 12: Model of the pyramidal antenna, with the gain as an overlay.

From the gain visualization, this is a lot more like what we'd expect and want, compared to my experiments with the cone antenna. Zooming in on the directivity plot, there is a back lobe and some side lobes present, but they are significantly less prominent than the main lobe, where the power density is the highest. In other words, almost all of the radiation this antenna would receive would be from the direction the antenna is pointed at, not from radiation in other directions. This matches what we expect to see for a horn antenna, so this antenna design should be good for what we want to do!

Plotting the S parameter for this antenna tells us what frequency or frequencies the antenna will be the most sensitive for. For this antenna, it seems like it is actually not the most sensitive at 20 GHz, but rather around 19.1 GHz. If we wanted to get closer to 20 GHz exactly, we could adjust the antenna's dimensions more, but this is still within the range we

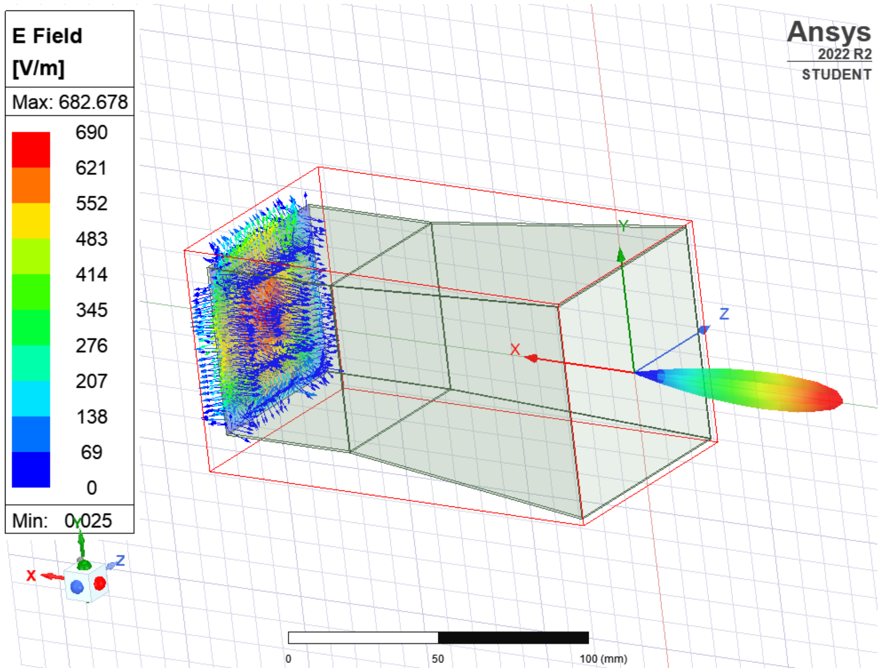


Figure 13: Model of the pyramidal antenna, with the gain as an overlay. This also shows the strength of the E field on the receiver from a far-field signal.

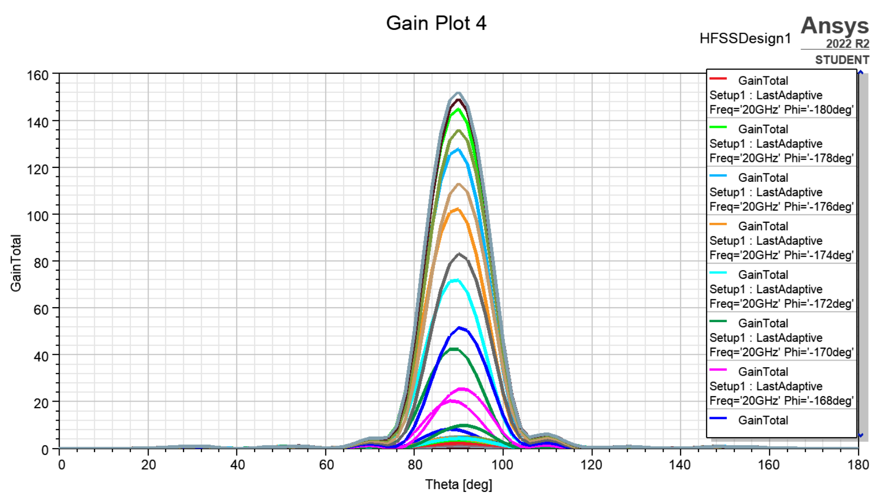


Figure 14: The plot of our gain has a clear, distinct peak in the direction $\theta = 90°$.

care about, and not too close to any emission or absorption lines that could mess with our measurements, so it should be good enough.

# 3 Limitations of This Design

Despite the attempts to account for noise and environmental factors, this setup will still have many limitations. Although we have a method for calibrating the electronics at different temperatures, it would be better if we could get the temperature even lower, to match the sky: right now, we are relying on extrapolating from the temperature of the coldest source

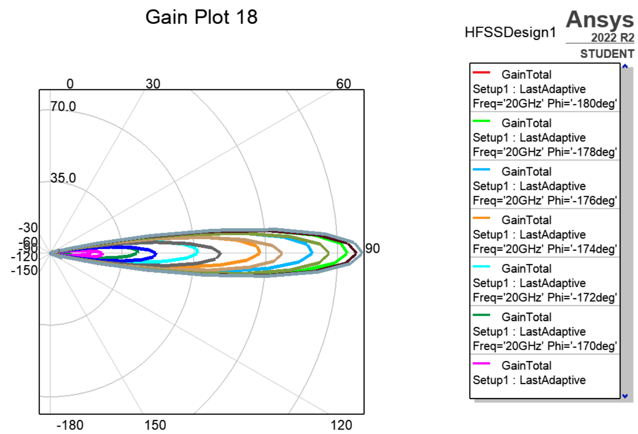


Figure 15: Polar plot of the radiation pattern, showing again the clear main lobe!

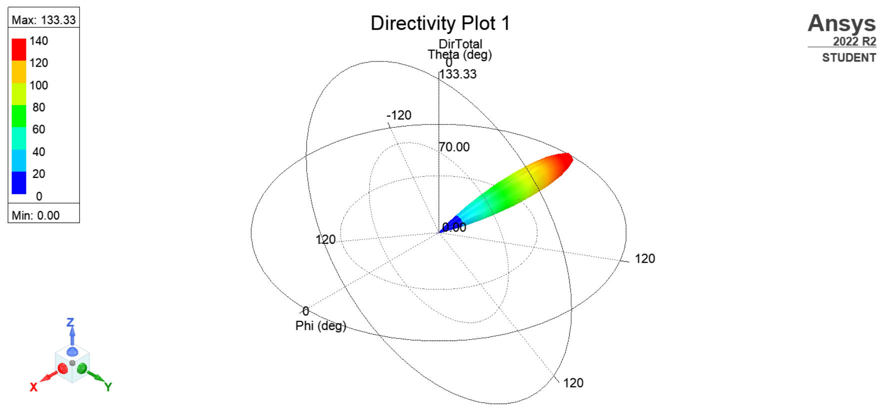


Figure 16: 3D directivity plot for this pyramidal antenna.

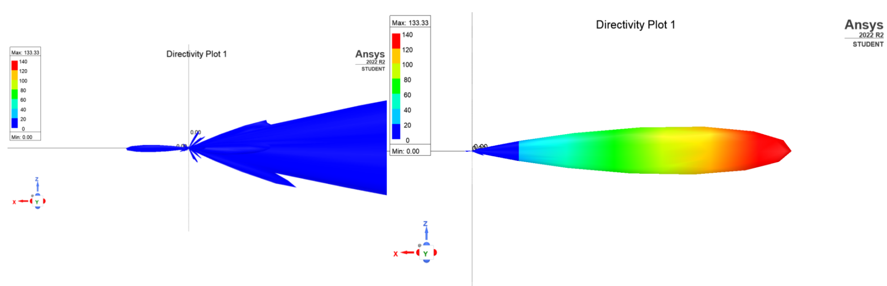


Figure 17: Zooming in on the directivity plot, showing a back lobe and side lobes that are significantly smaller than the main lobe.

we can reasonably find. To further minimize noise contributions from the electronics we could also cool them more. Attempting to observe the CMB through the atmosphere also introduces significant error as the atmosphere is not uniform and our measurements could be very sensitive to weather, humidity, and other atmospheric phenomena. To deal with both of these issues, then, if we wanted the best detector possible, we would fly a CMB detector on a high-altitude balloon or on a satellite. Plus, all of this is not accounting for the limitations of our electronics. Needing to convert the frequency down for using the RTL-

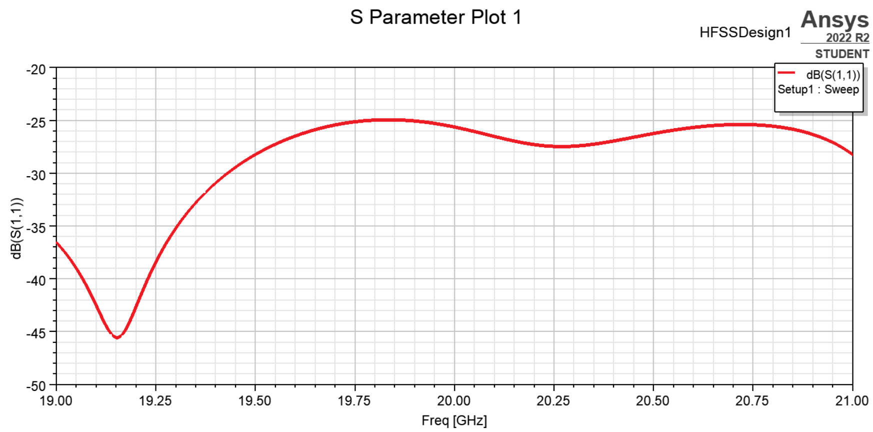


Figure 18: S-parameter for this antenna.

SDR also introduces possible error; the RTL-SDR's performance is not comparable to an actual high-quality spectrum analyzer, and realistically we may need even more filtering and amplification than what we have right now.

# 4 Historical Comparison

After this design process, I was curious about how my proposed instrument design compares to the instrument used in the first detection of the CMB. Overall, I was pretty surprised to see that in terms of design principles and overall elements, some of the broad strokes of my design and process were close to how Penzias and Wilson first found and measured the temperature of the CMB. Penzias and Wilson's original paper reporting their process and findings is short, but essentially, they had a microwave radiometer with a horn-reflector antenna.[8] This type of antenna is basically a composite of a horn antenna and a parabolic dish antenna: it has a waveguide section and a pyramidal horn, with a parabolic reflector mounted over the aperture of the horn. The advantage of this compared to having only a horn is that the parabolic surface can focus radiation into the horn section, have an improved gain compared to a standard horn, and operate with lower noise.[9] Penzias and Wilson's microwave radiometer was cooled with liquid helium to keep noise as low as possible. They accounted for atmospheric absorption by also sweeping the antenna over a range of angles and recording the change in temperature. From reading their report, I noticed there were factors I did not properly consider that would have significant effects on measuring power that they had to account for, including return losses, impedance matching, and measuring the power of the back and side lobes of the antenna's radiation pattern.[10]

[8]R. W. Penzias A. A.; Wilson. "A Measurement of Excess Antenna Temperature at 4080 Mc/s". In: *Astrophysical Journal* 142.3 (1965), pp. 419–421.

[9]D. C.; Hunt L. E. Crawford A. B.; Hogg. "A Horn-Reflector Antenna for Space Communication". In: *The Bell System Technical Journal* 40.4 (1961), pp. 1095–1116.

[10]Penzias, "A Measurement of Excess Antenna Temperature at 4080 Mc/s".

# 5 Conclusion

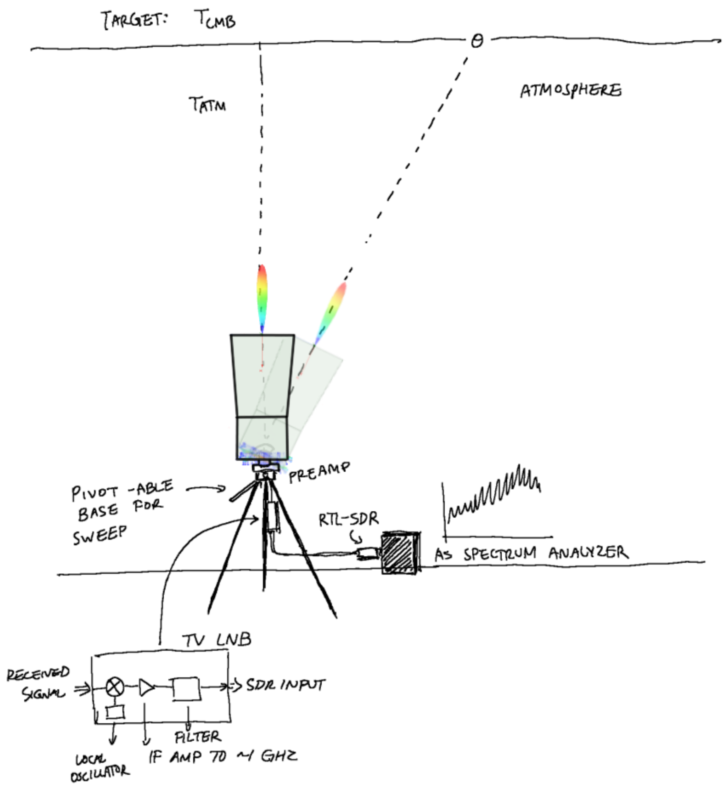


Figure 19: Here we are.

All in all, even though I didn't get to build this design and test out this process, I learned a lot through this project. The process of designing an antenna gave me a lot of insight about how RF electronics work both in theory and practice; this was my first time setting up an RF simulation and seeing how I could use this tool to design hardware was very interesting. Comparing what I considered for background and noise sources, and how I chose to account for them, with all the contributions, noise sources, and losses Penzias and Wilson actually had to characterize and eliminate was also insightful, and helped me develop a much better appreciation of the challenges in designing instruments for conducting cosmology research. In the process of working on this design, I found several examples of using the RTL-SDR and similar hardware for radio astronomy,[11] but no comparable examples of using it to detect and measure the CMB. Perhaps this is because of the sensitivity or resolution needed being a lot higher than what this hardware is capable of, or the difficulty in constructing and properly calibrating and characterizing such a setup. I would like to actually construct this instrument eventually, see how well commercially available electronics perform, and find out if my design works. It will be interesting as a proof of concept and for testing the limits of what readily available amateur radio hardware can do, that's for sure.

[11] *"SDR for Budget Radio Astronomy"*. 2017. URL: https://www.rtl-sdr.com/rtl-sdr-for-budget-radio-astronomy/.

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